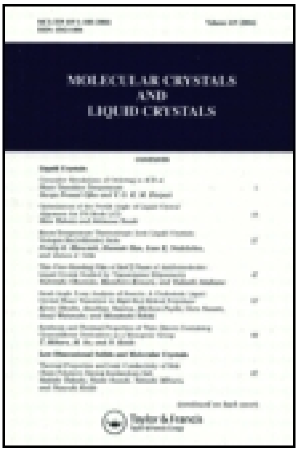

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Freedericsz Transition Dynamics in a Nematic Layer with a Surface Viscosity

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A simple model for the one-dimensional nematic director configuration is used for a qualitative description of orientation dynamics in a nematic layer with a surface viscosity. This parameter may appear in dependencies of transient times on the layer thickness. The approximate formulae for both the decay and rise times are derived.

INTRODUCTION

In general the Freedericsz transition dynamics are well studied both theoretically and experimentally.[1–14] In the continuum theory of a small-angle deformed nematic layer with a hard surface anchoring, the director orientation times are proportional to the rotational viscosity coefficient, γ_1, and inversely proportional to the curvature elastic constant, K. In the case of an electrically controlled cell, the rise time, t_r, is also inversely proportional to the normalized applied voltage, $U^2/U_c^2 - 1$, where U_c is the threshold. Both the rise time and the decay time, t_d, are proportional to the square of the thickness, L^2. Therefore the extrapolation to the vanishing thickness must result in vanishing director transient times. Of course, in the scope of the continuum theory, a layer thickness is implied greater than a parameter order length.[15–17]

The decay time is not altered by surface director pre-tilting to first order for a small pre-tilt angle.[3] On the other hand, derivation has shown that the weak anchoring leads to a linear component in the thickness dependence of the decay time in addition to a quadratic part,[3–5] and the extrapolated time mentioned above remains zero. However, this dependence was obtained with a vanishing surface energy dissipation, and it is of interest how a surface viscosity may perturb the transient times. An attempt to discuss this question is the subject of the paper.

APPROXIMATE DYNAMIC EQUATIONS

Let us consider the dynamics of a director tilt angle, θ, in a nematic layer perpendicular to the z axis, and use the equation

$$\gamma_1(\partial\theta/\partial t) = (\partial/\partial z)[(\partial/\partial\theta_{,z})F] - (\partial/\partial\theta)F, \tag{1}$$

where γ_1 is a rotational viscosity coefficient, F is a free energy density of the director in a volume, t is a current time, a comma denotes a spatial derivative, and back flow[1] is neglected.

The solution of Equation (1) can be obtained, in particular, as an "approximation with exact boundary conditions"[18] in the form

$$\theta(t,z) = \sum_n \theta_n(t) v_n(z) + \theta_s(t,z), \tag{2}$$

where the functions v_n, θ_s satisfy the follow boundary conditions at the layer surfaces $z = \pm L/2$:

$$v_n(\pm L/2) = 0, \quad \theta_s(t, \pm L/2) = \theta(t, \pm L/2). \tag{3}$$

Equation (1) multiplied by "homogeneous mode" $v_n(z)$ and then integrated over a layer thickness leads to the set of equations

$$\sum_m \left[(d\theta_m/dt) \int \gamma_1 v_m v_n dz \right] + \int \gamma_1 (\partial\theta_s/\partial t) v_n dz = -(\partial/\partial\theta_n) \int F dz. \tag{4}$$

To derive the right hand side of Equation (4), the identity followed from Equation (2)

$$\partial F/\partial\theta_n = (\partial F/\partial\theta) v_n + (\partial F/\partial\theta_{,z})(\partial v_n/\partial z),$$

has been used.

When functions $v_n(z)$ are mutually orthogonal the relationships

$$\int v_m v_n dz = \delta_{mn} N_n L \tag{5}$$

take place. Then the dynamic Equations (4) reduce to the equations set

$$L N_n \gamma_1 (d\theta_n/dt) + \gamma_1 \int (\partial\theta_s/\partial t) v_n dz = -(\partial/\partial\theta_n) \int F dz. \tag{6}$$

Imposing a linear dependence of $\theta_s(t, z)$ on z we have

$$\begin{aligned} & L N_n \gamma_1 (d\theta_n/dt) + (\gamma_1/2)(d\theta_+/dt + d\theta_-/dt) \int v_n dz \\ & + \gamma_1 (d\theta_+/dt - d\theta_-/dt) \int z v_n dz/L = -(\partial/\partial\theta_n) \int F dz, \end{aligned} \tag{7}$$

where the surface tilt angles, $\theta_\pm = \theta(t, \pm L/2)$, appear. It is necessary to know the surface tilt angle velocities in Equation (7). They are provided by the boundary conditions[19]

$$\gamma_s^\pm (d\theta_\pm/dt) = -dI^\pm/d\theta_\pm \mp K_{11}(1 + \delta K \sin^2\theta_\pm) \theta_{\pm,z}, \tag{8}$$

where $\gamma_s^{\pm}$ are surface viscosity coefficients at layer boundaries $z = \pm L/2, I^{\pm}(\theta_{\pm})$ are surface anchoring energy functions, K_{11} is a splay elastic constant, $\delta K = (K_{33} - K_{11})/K_{11}$, K_{33} is a bend constant.

In the case of a transverse electric field in a layer, the free energy integral is

$$f_L = \int F dz = (K_{11}/2)\int(1 + \delta K \sin^2\theta)\theta_{,z}^2 dz - \varepsilon_\perp U^2 \Big/ \left(8\pi \int \beta dz\right), \tag{9}$$

where U is a control voltage, $\beta = 1/(1 + \delta\varepsilon \sin^2\theta)$, $\delta\varepsilon = (\varepsilon_\parallel - \varepsilon_\perp)/\varepsilon_\perp$, and $\varepsilon_\parallel, \varepsilon_\perp$ are permittivities parallel and perpendicular to the director.

Using dimensionless quantities for all the coordinates, $\zeta = z/L$, the control voltage, $u = U/(4\pi^3 K_{11}/|\varepsilon_\parallel - \varepsilon_\perp|)^{1/2}$, and the time, $\tau = t/(\gamma_1 L^2/\pi^2 K_{11})$, one transforms the Equations (7), (9) into the form

$$N_n(d\theta_n/d\tau) + (1/2)(d\theta_+/d\tau + d\theta_-/d\tau)\int v_n d\zeta$$
$$+ (d\theta_+/d\tau - d\theta_-/d\tau)\int \zeta v_n d\zeta = -\partial\Phi/\partial\theta_n, \tag{10}$$

$$\Phi = (1/2)\int(1 + \delta K \sin^2\theta)(\theta_{,\zeta}/\pi^2)^2 d\zeta - u^2 \Big/ \left(2\delta\varepsilon \int \beta d\zeta\right). \tag{11}$$

For the set of harmonics

$$v_{2k-1}(\zeta) = \cos[(2k-1)\pi\zeta], \quad v_{2k}(\zeta) = \sin[2k\pi\zeta] \tag{12}$$

with $k = 1,2,3,\ldots$, the normalizing factor in conditions (5) is 1/2, and the Equations (10) reduce to

$$d\theta_{2k-1}/d\tau = (-1)^k/(2k-1)(2/\pi)(d\theta_-/d\tau + d\theta_+/d\tau) - 2(\partial\Phi/\partial\theta_{2k-1}), \tag{13'}$$

$$d\theta_{2k}/d\tau = (-1)^{k+1}/(2k)(2/\pi)(d\theta_-/d\tau - d\theta_+/d\tau) - 2(\partial\Phi/\partial\theta_{2k}). \tag{13''}$$

Using the Rapini form of the surface anchoring energy

$$I^{\pm} = (W^{\pm}/2)\sin^2(\theta_{\pm} - \theta^{\pm}), \tag{14}$$

where $\theta^{\pm}$ are pretilted surface director angles, we have from Equation (8):

$$d\theta_{\pm}/d\tau = \{-(\gamma_1 L/\pi\gamma_s)((1/2\lambda)\sin[2(\theta_{\pm} - \theta^{\pm})] \pm (1 + \delta K \sin^2\theta_{\pm})(\theta_{\pm,\zeta}/\pi))\} \tag{15}$$

Here $\lambda^{\pm} = (\pi K_{11}/W^{\pm}L)$ is an anchoring parameter for a homogeneous cell.

Therefore, in consequence of the approximation (2) and assumptions (3) and (12), the orientation dynamics in a nematic layer is given by Equations (11), (13), (15).

ESTIMATION OF TRANSIENT TIMES

For the sake of a simple, qualitative analysis, only the first term of the expansion (2) need be kept. Then, in the case of identical pretilt angles at both surfaces fencing a layer, the director configuration is given roughly by

$$\theta(\tau, \zeta) = \theta_1 \cos \pi\zeta + \theta_-(\tau) \tag{16}$$

In fact, such a qualitative approximation, with the exception of the dependence θ_- on τ, was used elsewhere[20] for example. Moreover, we take uniconstant elastic approximation, $\delta K = 0$, and small director angles, $\theta < 1$. In these circumstances the problem in Equations (11), (13), and (15) reduces to

$$(d/d\tau)(\theta_- + \pi\theta_1/4) = u^2(\theta_- + \pi\theta_1/4) - \pi\theta_1/4, \tag{17'}$$

$$\gamma^*(d\theta_-/d\tau) = \theta_1 - (\theta_- - \theta^-)/\lambda, \tag{17''}$$

where $\gamma^* = \pi\gamma_s/L\gamma_1$.

The Equation (17′) shows that in the above approximation, the characteristic director angle, $\theta^* = \theta_- + \pi\theta_1/4$, determines a nematic orientation in a layer as a whole. Therefore, there is a need to study the problem

$$d\theta^*/d\tau = (u^2 - 1)\theta^* + \theta_-, \tag{18'}$$

$$\gamma^*(d\theta_-/d\tau) = (4/\pi)(\theta^* - \theta_-) - (\theta_- - \theta^-)/\lambda. \tag{18''}$$

The case of a vanishing surface viscosity. This case has been worked out previously, and Equations (18) with $\gamma^* = 0$ must lead to known results. Excluding θ_- from (18) one has

$$d\theta^*/d\tau = (u^2 - u_c^2)\theta^* + u_c^2\theta^-, \tag{19}$$

where $u_c = (1 + 4K/WL)^{-1/2}$. When θ^- is equal to zero, the modulus of θ^* increases if $u > u_c$. Therefore u_c is an approximate threshold voltage. The exact value of a threshold voltage, h', is determined by the equation[14]

$$\lambda h' = ctg(\pi h'/2).$$

From this equation $h' = (1 + 2K/WL)^{-1}$ for small surface anchoring lengths, $b = k/W$, and $h' = (2WL/\pi^2 K)^{1/2}$ for large anchoring lengths. It is easy to see that u_c can be used as an acceptable approximation to h'.

Taking $\theta_0^* > \theta^-$ at $\tau = 0$ and $u^2 = 0$ one studies the decay by the solution of the Equation (19)

$$\theta^* - \theta^- = (\theta_0^* - \theta^-)\exp(-u_c^2\tau),$$

where the decay time is $\tau_d = 1/u_c^2$, or in usual units is

$$t_d = (\gamma_1 L^2/\pi^2 K)(1 + 4K/WL). \tag{20}$$

In the case of a small length $b = K/W$ the expression (20), to first order in b, is analogous to the formula reported elsewhere[3]

$$\sigma_1 = \gamma_1(d + 2b)^2/\pi^2 K_{33}, \tag{21}$$

were σ_1 denotes a decay time, d is a thickness , $b = K_{33}/C$, C is an anchoring energy.

For large anchoring length the dependence (20) of t_d on b is favoured.

To study the rise time, one takes in Equation (19) $\theta_0^* = \theta^-$ and $u^2 > u_c^2$. Then the solution can be written in the form

$$\theta^* = (\theta^-/[u^2 - u_c^2])([u^2 \exp([u^2 - u_c^2]\tau) - u_c^2).$$

Therefore, the rise time is

$$t_r = (\gamma_1 L^2/\pi^2 K)/(u^2 - u_c^2). \tag{22}$$

This formula is also well known.[1-3]

The case of a non-zero surface viscosity. In this case, in order to eliminate θ_- from Equations (18), one can rewrite the Equation (18″) in the form

$$\theta_- = A + D\theta_- = A + D(A + D(A + D(...\theta_-))) = A + \sum_{k=1}^{N} D^k A + D^{N+1}\theta_-, \tag{23'}$$

were $A = (1 - u_c^2)\theta^* + u_c^2\theta^-$ and D is the differential operator, $(-\gamma^*\lambda u_c^2)(d/d\tau)$. Therefore, if $D^N\theta^*$ vanishes with increasing N we obtain the equation

$$\theta_- = (1 - u_c^2)\theta^* + u_c^2\theta^- + (1 - u_c^2) \sum_{k=1} (d^k\theta^*/d\tau^k)(-\gamma^*\lambda u_c^2)^k. \tag{23''}$$

By using this relationship in Equation (18′), we can make the dynamic equation for small $\gamma^*\lambda u_c^2$:

$$(d\theta^*/d\tau)(1 + \gamma^*\lambda u_c^2[1 - u_c^2]) = (u^2 - u_c^2)\theta^* + u_c^2\theta^-. \tag{24}$$

It is clear from this equation that the director configuration transient times can be found along the same line as in (19) and they are given in the form

$$t' = (\gamma_1 L^2/\pi^2 K)(1 + \gamma^*\lambda u_c^2[1 - u_c^2])/(u^2 - u_c^2). \tag{25}$$

The decay time deduced from this by $u^2 = 0$ is

$$t_d' = (\gamma_1 L^2/\pi^2 K) + (4/\pi^2)(\gamma_1 L/W) + (\gamma_s/W)([4K/WL]/[1 + 4K/WL]). \tag{26}$$

The rise time is

$$t_r' = (\gamma_1 L^2/\pi^2 K + (\gamma_s/W)[4K/WL]/[1 + 4K/WL]^2)/(u^2 - u_c^2). \tag{27}$$

In the particular case of $\gamma_s = 0$ these expressions include old ones, (20) and (22). Equation (26) shows that a surface viscosity leads to the non-zero extrapolated decay time, $t'_d = \gamma_s / W$, by extrapolation of the thickness to zero, especially for small W. Equation (27) shows that the rise time tends to a linear function of the thickness, $t'_r = \gamma_s L/4K(u^2 - u_c^2)$, for a small thickness, especially for small K.

DISCUSSION

Thus, in the case of weak surface anchoring with an energy dissipation, in general there is a non-zero extrapolated value of the director orientation decay time in the thickness dependence. It leads to more expressive behaviour of the optical response determined by a change of an optical path difference. If the nematic optical anisotropy is Δn, then the path difference roughly is $\Delta n L \cos^2\theta^* \cong \Delta n\, L(1 - \theta^{*2})$ for small Δn and θ^*. Then the relaxation from the state of θ_p^* with changing of the path differences by Δ_p takes place in the time of $t_p = (t'_d/2)(-\ln[1 - \Delta_p/\Delta n\, L\theta_p^{*2}])$. From this last expression, it is clear that the optical switch-off time may even increase with decreasing thickness. This is reminiscent of the case of a hard director deformation regime with rigid anchoring, when the optical switching times also increase with decreasing thickness and remain constant or increase with increasing thickness[9,11,12]. In conclusion let us speculate that the considered influence of a surface viscosity is qualitatively the same in other geometries not only a nematic planar layer.

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